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MULTIPLY PERFECT NUMBERS.

BY DERRICK N. LEHMER.

1. By a *perfect number* is meant a number which is equal to the sum of those of its divisors which are less than the number. Thus $6 = 1 + 2 + 3$ is a perfect number.

We shall define a *multiply perfect number* as one which is an exact divisor of the sum of all its divisors; the quotient being the *multiplicity*. Thus ordinary perfect numbers like 6 or 28 have the multiplicity 2.

The existence of multiply perfect numbers of higher multiplicity than 2 is shown by the following list.

NUMBER.	MULTIPLICITY.
$2^3 \cdot 3 \cdot 5$	3
$2^5 \cdot 3 \cdot 7$	3
$2^9 \cdot 3 \cdot 11 \cdot 31$	3
$2^{13} \cdot 3 \cdot 11 \cdot 43 \cdot 127$	3
$2^{14} \cdot 5 \cdot 7 \cdot 19 \cdot 31 \cdot 151$	3
$2^3 \cdot 3^2 \cdot 5 \cdot 7 \cdot 13$	4
$2^2 \cdot 3^2 \cdot 5 \cdot 7^2 \cdot 13 \cdot 19$	4
$2^5 \cdot 3^3 \cdot 5 \cdot 7$	4
$2^7 \cdot 3^3 \cdot 5^2 \cdot 17 \cdot 31$	4
$2^8 \cdot 3 \cdot 5 \cdot 7 \cdot 19 \cdot 37 \cdot 73$	4
$2^8 \cdot 3^2 \cdot 7^2 \cdot 13 \cdot 19^2 \cdot 37 \cdot 73 \cdot 127,$	4
$2^9 \cdot 3^2 \cdot 7 \cdot 11 \cdot 13 \cdot 31,$	4
$2^{14} \cdot 3 \cdot 5 \cdot 7 \cdot 19 \cdot 31 \cdot 151,$	4
$2^7 \cdot 3^4 \cdot 5 \cdot 7 \cdot 11^2 \cdot 17 \cdot 19,$	5
$2^{10} \cdot 3^5 \cdot 5 \cdot 7^2 \cdot 13 \cdot 19 \cdot 23 \cdot 89,$	5
$2^{11} \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 13 \cdot 19 \cdot 31,$	5
$2^{21} \cdot 3^6 \cdot 5^2 \cdot 7 \cdot 19 \cdot 23^2 \cdot 31 \cdot 79 \cdot 89 \cdot 137 \cdot 547 \cdot 683 \cdot 1093,$	5
$2^{20} \cdot 3^3 \cdot 5 \cdot 7^2 \cdot 13^2 \cdot 19 \cdot 31 \cdot 61 \cdot 127 \cdot 337,$	5
$2^{19} \cdot 3^6 \cdot 5^3 \cdot 7^2 \cdot 11 \cdot 13 \cdot 19 \cdot 23 \cdot 31 \cdot 41 \cdot 137 \cdot 547 \cdot 1093,$	6
$2^{24} \cdot 3^8 \cdot 5 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19^2 \cdot 31 \cdot 43 \cdot 53 \cdot 127 \cdot 379 \cdot 601 \cdot 757 \cdot 1801,$	6

The numbers in the above list were all obtained by purely tentative methods. No formula for multiply perfect numbers has been obtained except the well-known one for those of multiplicity 2 (ordinary perfect numbers), namely, $2^{i-1}(2^i-1)$, where 2^i-1 is prime.*

2. If $\kappa = \prod\limits_{i=1}^{r} p_i^{a_i}$ is a perfect number of multiplicity n, then we have

$$n\kappa = \prod_{i=1}^{r} \sum_{j=0}^{a^i} p_i^j = \prod_{i=1}^{r} \frac{p_i^{a_i+1}-1}{p_i-1}$$

or

$$n = \prod_{i=1}^{r} \frac{p_i - \dfrac{1}{p_i^{a_i}}}{p_i-1};$$

so that

$$n < \prod_{i=1}^{r} \frac{p_i}{p_i-1}. \tag{1}$$

Sylvester† has derived this formula for the case $n = 2$. It is of fundamental importance in his proof of the non-existence of odd perfect numbers containing only three distinct prime divisors.

3. The object of the present note is a generalization of Sylvester's results for the case of multiply perfect numbers.

Multiply perfect numbers of multiplicity 3, *containing less than three distinct primes, do not exist.* For, from (1) it follows that $3 < \prod\limits_{i=1}^{r} \frac{p_i}{p_i-1}$, and the greatest value which this product can have when there are only two elements is $\frac{2}{1} \cdot \frac{3}{2}$, which is not greater than 3.

Similarly: *Multiply perfect numbers of multiplicity* 4 *must contain at least* 4 *distinct prime divisors; those of multiplicity* 5, *at least* 6 *distinct prime divisors; those of multiplicity* 6, *at least* 9 *distinct prime divisors; those of multiplicity* 7, *at least* 14 *distinct prime divisors; etc.*

BERKELEY, CAL., JANUARY, 1900.

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* Given by Euclid and shown by Euler to include all even perfect numbers. *Cf. Encyclopädie der mathematischen Wissenschaften*, vol. 1, art. C 1, Bachmann.

† *Comptes Rendus*, vol. 106 (1888), pp. 403, 446, 522, 641.